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**Particle in a Finite Square Well Worksheet**

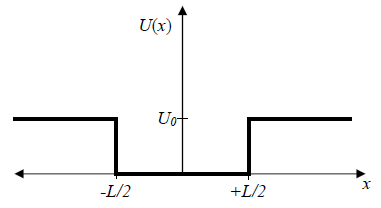
**Physics 205**

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(Adopted from M. Trawick)

Previously we looked at the case of a particle that was stuck inside a one-dimensional “box,” or “tube.” Outside of this box, the potential energy U(x) was infinite. This meant that the electron (or whatever) absolutely positively could not get out of the box, no matter how big its energy was. These “boxes” are also called “infinite potential wells” since the potential energy goes up to U = at the edges and beyond.

Now we’ll play with potential wells that are finite, in the sense that the potential energy outside the edges does not go to infinity. The graph below shows an example



Okay, so let’s see what the Schrödinger equation has to tell us about the wave function for a particle inside a one-dimensional finite square well. The Schrödinger equation says that

where U(x) = 0 inside the well, and U(x) = U0 outside the well. Inside the well, we already know

the answer: the Schrödinger equation reduces to

with solutions

where

1. Now, let’s look at *outside* the box, say for x > L/2 . Here, we have to keep U(x) in the Schrödinger equation:

But now the constant E-U0 is itself negative, so we are now looking for a function where if you take the second derivative of it, you get a positive constant times that same function. What are two possible functions that fit the bill? (They differ only by a strategically placed negative.)

2. Of the two equations you have written above for , one works great for x < -L/2 , one works for x > L/2 . Which is which?

In the region between the walls the function will have to be made out of sines and cosines, and it will have to be continuous with the function outside the walls, as we’ll discuss. It is the region within the walls and the need to be continuous that will set up quantization so that the wavefunction will depend on *n* with only integer values of *n* allowed.

To give yourself some sense of what looks like for different values of energy E, open the following page in Internet Explorer:

<http://webphysics.davidson.edu/physletprob/ch10_modern/default.html>

Click on “Finite Well” on the left hand side. This simulation shows a 1-dimensional box running from x =-0.50 to x = +0.50. The potential energy is shown as a red line at U = 0, with the “infinite” vertical walls just off screen. Initially, your particle has been given an energy of E = 8.712 (in some units), which leads to the ground state (*n*=1) function shown in blue.

3. Draw sketches of below for E = 34 and E = 35 (Type the numbers in the Energy box and hit enter). Are either one of these a good-looking wave function for ? Why or why not?

4. Now try some values between E = 34 and E = 35. What’s the best value for the second energy state? Check your answer by setting *n* = 2 and clicking "find."

5. Notice that the energy well has a height of U0 = 1000 in some units. Draw a sketch of the wave function when you set E = 1500. In this case, is the particle “free” to leave the well, or is it “bound” inside of the well?

6. How many “bound” states are there for this particle in this finite potential well? Remember that the line is moving upward as *n* increases in this graphical visualization.

7. Classically, if you had a bowling ball in a valley between two hills, you would think that if the height of the potential well is U0 = 1000, and the bowling ball has a total energy of E = 700, then there would be zero probability of ever finding the bowling ball located outside of the well, at x > L/2. Look carefully at the wave function for *n* = 9. According to this simulation, is there a zero probability of finding the particle at x > L/2 here?

8. Which is greater: the probability of finding the particle at x > L/2 if it is in the *n* = 9 state, or the probability of finding the particle at x > L/2 if it is in the *n* = 1 state? Sketch the graphs to show why.

9. Suppose the particle is in the *n* = 6 state. Which is greater: the probability of finding the particle at x > L/2 or the probability of finding the particle at x < -L/2, or are they the same? (Sketch the graphs to be sure.)